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Heat transfer in a rarefied polyatomic gas—II. Sphere

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Abstract—The Hanson–Morse model of the linearized Wang Chang–Uhlenbeck equation is used to calculate the heat transfer from a spherical particle situated in an infinite expanse of a polyatomic gas. Results for heat transfer, density, and internal and translational temperature profiles for all degrees of rarefactions and for arbitrary internal and translational accommodation coefficients are obtained. Also studied is the dependence of these results on Knudsen number, internal energy, Eucken factor, and collision relaxation number.

INTRODUCTION

THE PROBLEM of heat transfer from a sphere (or between concentric spheres) in a rarefied gas has been studied both experimentally and theoretically by a number of investigators. For monatomic gases, the available work includes the theoretical investigations of Takao [1], Lees and Liu [2], Springer and Tsai [3], Cercignani and Pagani [4], and the experimental measurements of thermal accommodation coefficients of helium, argon and xenon gases on Zircaloy-2 and UO_2 spheres at room temperature in refs. [5, 6]. However, to our knowledge, no commensurate theoretical or experimental investigations have been conducted on heat transfer from a sphere in rarefied polyatomic gases.

In this paper we extend our previous work on heat transfer in a rarefied polyatomic gas between two parallel plates [7] to heat transfer from a spherical particle situated in an infinite expanse of a polyatomic gas. In addition to heat transfer calculations, density, translational and internal temperature profiles are calculated for all ranges of Knudsen number and for arbitrary translational and internal accommodation coefficients. The dependence of the above on four dimensionless parameters describing the polyatomic gas is studied. These parameters are: (1) G, the dimensionless, constant volume, internal heat capacity; (2) Z, the collisional relaxation number; (3) the total Eucken number, f_t; and (4) the Knudsen number.

The problem is solved using the Hanson–Morse model of the linearized Wang Chang–Uhlenbeck equation (hereafter referred to as WCU equation), as given by Hanson and Morse [8], and Cipolla [9]. We convert the relevant integro-differential equation with associated boundary conditions into a system of integral equations, which is then solved by a numerical technique.

STATEMENT OF THE PROBLEM

Consider a sphere of non-dimensional radius r_0 at rest in an infinite expanse of a polyatomic gas. Here $r_0 = \tilde{r}_0/l$, where $\tilde{r}_0$ is the radius of the sphere and l is the mean free path. The surface of the sphere is maintained at a constant temperature T_0, different from the gas temperature T_∞ at large r. Let $\mathbf{r}$ and $\mathbf{c}$ be, respectively, the non-dimensional position and velocity vectors of a gaseous molecule. If $\Delta T/T_\infty = (T_0 - T_\infty)/T_\infty$ is small, the WCU equation for the distribution function $f(\mathbf{r}, \mathbf{c}, E_i)$ can be linearized by writing $f = f_\infty(1+h)$. Here h is a measure of the perturbation on the distribution function from the local Maxwellian, $|h| \ll 1$.

Assuming the Hanson–Morse model, the steady-state form of the linearized WCU equation in the absence of external forces leads to the following non-dimensional boundary-value problem:

$$\mathbf{c} \cdot \frac{\partial h}{\partial \mathbf{r}} = Lh \tag{1}$$

where

$$Lh = -h(\mathbf{r}, \mathbf{c}, \varepsilon_i) + \sum_{m=1}^{5} \psi_m(\mathbf{c}, \varepsilon_i; r) a_m(r)$$

with boundary conditions

$$h^+ = \gamma + \tau_{\mathrm{tr}}(c^2 - 2) + \tau_{\mathrm{int}}(\varepsilon_i - G), \quad r \in \partial R, \quad \mathbf{c} \cdot \mathbf{n}_r > 0 \tag{2}$$

$$h(\mathbf{r}, \mathbf{c}, \varepsilon_i) \to 0 \quad \text{for } |\mathbf{r}| \to \infty \tag{3}$$

where the functions ψ_m are given by

$$\psi_1 = 1 \tag{4}$$

$$\psi_2 = [(2/3)(c^2 - 3/2)(1 - (2G/3Z)) + (2/3Z)(\varepsilon_i - G)] \tag{5}$$

NOMENCLATURE

$\tilde{\mathbf{c}}$ dimensional velocity
$\mathbf{c}$ dimensionless molecular velocity, $c = \tilde{c}(2RT_0)^{-1/2}$
c_v^i internal specific heat per molecule
c_v^{tr} translational specific heat per molecule
c_v total specific heat, $(2/3)k + c_v^i$
E_i internal energy of level i (dimensional)
f_i molecular velocity distribution for particles in level i
f_{0i} absolute Maxwell–Boltzmann distribution
f_t, f_{tr}, f_{int} total, translational and internal Eucken numbers, respectively
G dimensionless specific heat
h perturbation of distributions
k Boltzmann constant
l mean free path
m molecular mass
n_∞ number density far away from the sphere
p_∞ gas pressure, $n_\infty k_0 T_\infty$
q total heat flux in r-direction
$q_{tr}(r)$ translational heat flux in r-direction
$q_{int}(r)$ internal heat flux in r-direction
$q_{fm}(r)$ free molecular heat flux in r-direction
Q partition function, $\Sigma \exp(-E_i/kT)$
R gas constant
$\tilde{r}_0$ radius of the sphere (dimensional)
r_0 inverse Knudsen number, $\tilde{r}_0/l$
T_0 surface temperature of the sphere
T_∞ temperature of the gas far away from the sphere
T_{tr}, T_{int} translational and internal temperature, respectively
Z collision relaxation number.

Greek symbols
α_{tr} translational accommodation coefficient
α_{int} internal accommodation coefficient
ε_i dimensionless internal energy
λ, λ_{tr}, λ_{int} total, translational and internal thermal conductivities, respectively
μ gas viscosity
τ_c inverse of the total collision frequency
τ_r relaxation time for the internal degree of freedom.

$$\psi_3 = [(2/3)(c^2-3/2)(1/Z)+(1/G)(\varepsilon_i-G)(1-1/Z)] \quad (6)$$

$$\psi_4 = [(4/9)c_r(c^2-5/2)(1-G/Z)+c_r(\varepsilon_i-G)(2/3Z)] \quad (7)$$

$$\psi_5 = [c_r(c^2-5/2)(2/(3Z))+c_r(\varepsilon_i-G)(1-F)(2/G)]. \quad (8)$$

The moments $a_m(r)$ are expressed as

$$a_m(r) = ((\rho_m, h)) = \sum_i \int d\mathbf{c} \frac{\exp(-c^2-\varepsilon_i)}{Q_0 \pi^{3/2}} \times h(\mathbf{r}, \mathbf{c}, \varepsilon_i)\rho_m(\mathbf{c}, \varepsilon_i) \quad (9)$$

and the functions 'ρ' are defined as

$$\rho_1(\mathbf{c}, \varepsilon_i) = 1 \quad (10)$$

$$\rho_2(\mathbf{c}, \varepsilon_i) = c^2 - 3/2 \quad (11)$$

$$\rho_3(\mathbf{c}, \varepsilon_i) = \varepsilon_i - G \quad (12)$$

$$\rho_4(\mathbf{c}, \varepsilon_i) = c_r(c^2 - 5/2) \quad (13)$$

$$\rho_5(\mathbf{c}, \varepsilon_i) = c_r(\varepsilon_i - G). \quad (14)$$

The non-dimensional variables are defined as

$$r_0 = \frac{\tilde{r}_0}{l} \quad (15a)$$

$$c^2 = \frac{\tilde{c}^2}{2RT_\infty} \quad (15b)$$

$$\varepsilon_i = \frac{E_i}{kT_\infty} \quad (15c)$$

$$G = \frac{c_v^i}{k}. \quad (15d)$$

Moments $a_1(r)$ through $a_5(r)$, as calculated with equation (9), define, respectively, the density, translational and internal temperature perturbations, and the radial translational and internal heat flux as

$$a_1(r) = \frac{n(r) - n_\infty}{n_\infty} \quad (16a)$$

$$a_2(r) = \frac{T_{tr}(r) - T_{tr,\infty}}{T_{tr,\infty}} \quad (16b)$$

$$a_3(r) = \frac{T_{int}(r) - T_{int,\infty}}{T_{int,\infty}} \quad (16c)$$

$$a_4(r) = q_{tr}(r) \quad (16d)$$

$$a_5(r) = q_{int}(r) \quad (16e)$$

where q_{tr} and q_{int} are related to the actual dimensional radial fluxes by

$$\tilde{q}_{tr}(r) = p_\infty(2RT_\infty)^{1/2} q_{tr}(r) \quad (17)$$

$$\tilde{q}_{int}(r) = p_\infty(2RT_\infty)^{1/2} q_{int}(r). \quad (18)$$

Also the mean free path l is given by

$$l = (4/3)(\mu/p_\infty)(2RT_\infty)^{-1/2} \quad (19)$$

where μ is the gas viscosity coefficient. p_∞ and R are the density of the gas far away from the sphere and the gas constant, respectively. The collision relaxation number, Z, is defined by

$$Z = \tau_r/\tau_c = 3P_\infty \tau_r/2\mu \quad (20)$$

where τ_r, τ_c and p_∞ are relaxation time, mean collision time, and gas pressure far away from the sphere, respectively. The factor, F, is given by

$$F = \frac{(10/9)(G/Z)+(2G/3)(4/9+5G/9Z) + (5G/18Z^2)(c_v/k)f_t}{(4/9+5G/9Z)(c_v/k)f_t-(5/3)}. \quad (21)$$

The Eucken factor, f_t, is defined by

$$\lambda m/\mu = f_t c_v = f_{tr} c_v^{tr} + f_{int} c_v^{i} \quad (22)$$

where λ is the gas thermal conductivity coefficient; f_{tr} and f_{int} are the translational and internal Eucken factors; and c_v^{tr} is the translational heat capacity of the gas. Further, the constants γ, τ_{tr}, and τ_{int} of equation (2) are given by

$$\gamma = ((1,h^-))_b \quad (23)$$

$$\tau_{tr} = \alpha_{tr} + ((1-\alpha_{tr})/2)((c'^2-2,h^-))_b \quad (24)$$

$$\tau_{int} = \alpha_{int} + (1-\alpha_{int})(1/G)((\varepsilon_j - G, h^-))_b \quad (25)$$

where the scalar product is defined by

$$((f,h^-))_b = \frac{2}{\pi}\sum_j \frac{\exp(-\varepsilon_j)}{Q_0} \times \int \mathrm{d}\mathbf{c}' \exp(-c'^2)|\mathbf{c}'\cdot\mathbf{n}| f h^-(\mathbf{x},\mathbf{c}',\varepsilon_j),$$

$$x \in \partial R, \mathbf{c}'\cdot\mathbf{n} < 0. \quad (26)$$

Here α_{tr} is the accommodation coefficient for translational (tr) heat flux and α_{int} is the coefficient for internal (int) heat flux

$$\alpha_{tr} = \frac{\dot{q}''_{tr,in} - \dot{q}''_{tr,out}}{\dot{q}''_{tr,in} - \dot{q}''_{tr,m}} \quad (27)$$

$$\alpha_{tr} = \frac{\dot{q}''_{int,in} - \dot{q}''_{int,out}}{\dot{q}''_{int,in} - \dot{q}''_{int,m}} \quad (28)$$

where $\dot{q}''$ indicates heat flux, subscripts 'in' and 'out' indicate the inward and outward components, respectively, and subscript m indicates the outward flux assuming that the reflected (outward) molecules had a Maxwellian distribution corresponding to the temperature of the sphere.

ASYMPTOTIC SOLUTIONS

Continuum limit

In the continuum limit ($Kn \ll 1$), the solution to equation (1) is given by the classical Chapman–Enskog theory in the form

$$h_{asy}(\mathbf{c},\mathbf{r},\varepsilon_i) = \frac{T(\mathbf{r})-T_\infty}{T_\infty}((c^2-5/2)+(\varepsilon_i-G)) + \mathbf{c}\cdot\frac{\nabla T(\mathbf{r})}{T_\infty}A(\mathbf{c},\varepsilon_i) \quad (29)$$

where $A(\mathbf{c},\varepsilon_i)$ is the solution of the integral equation

$$L(A(\mathbf{c},\varepsilon_i)c_r) = c_r((c^2-5/2)+(\varepsilon_i-G)) \quad (30)$$

and L is the Hanson–Morse model operator. The function $A(\mathbf{c},\varepsilon_i)$ is given in terms of the model parameters by

$$A(\mathbf{c},\varepsilon_i) = W_1(c^2-5/2)+W_2(\varepsilon_i-G) \quad (31)$$

where constants W_1 and W_2 are defined as

$$W_1 = \frac{F+\dfrac{G}{3Z}}{\left(\dfrac{4}{9}+\dfrac{5G}{9Z}\right)F-\dfrac{5G}{18Z^2}} \quad (32)$$

and

$$W_2 = \frac{\left(\dfrac{4}{9}+\dfrac{5G}{9Z}\right)-\dfrac{5}{6Z}}{\left(\dfrac{4}{9}+\dfrac{5G}{9Z}\right)F-\dfrac{5G}{18Z^2}} \quad (33)$$

W_1 and W_2 are related as follows to the translational and internal thermal conductivities and Eucken factors:

$$f_{tr} = (mP_\infty/T_\infty)l(2RT_\infty)^{1/2}(5W_1)/(4\mu c_{vtr}) = -(10/9)W_1 \quad (34)$$

$$f_{int} = -(mP_\infty/T_\infty)l(2RT_\infty)^{1/2}(GW_2)/(2\mu c_v^{i}) = -(2/3)W_2. \quad (35)$$

Using equations (9), (13) and (14), the non-dimensional total heat flux in this limit is given as

$$q(r) = \frac{1}{T_\infty}\frac{\partial T(r)}{\partial r}\left[\frac{5}{4}W_1+\frac{W_2}{2}G\right]. \quad (36)$$

Free molecular limit

In this limit ($Kn \gg 1$), the distribution function h is given as

$$h = [\alpha_{tr}(c^2-2)+\alpha_{int}(\varepsilon_i-G)] \quad \text{for } \sin\theta \leqslant r_0/r. \quad (37)$$

Again using equations (9), (13) and (14), the total heat flux in this regime is

$$q_{fm}(r) = \frac{r_0^2}{\pi^{1/2}r^2}(\alpha_{tr}+(G/2)\alpha_{int}). \quad (38)$$

METHOD OF SOLUTION FOR ARBITRARY KNUDSEN NUMBER

Integration of equation (1) along the characteristic path, s, results in the expression

$$h(\mathbf{r}, \mathbf{c}, \varepsilon_i) = \int_0^\infty \frac{1}{c} \exp(-s''/c) \times \sum_{m=1}^{5} \psi_m(\mathbf{c}, \varepsilon_i; r') a_m(r')\, ds'' + h(\mathbf{r}_0, \mathbf{c}, \varepsilon_i) \exp(-s/c) \quad (39)$$

where, $\mathbf{r}' = \mathbf{r} - s''\mathbf{\Omega}$ and $s'' = |\mathbf{r} - \mathbf{r}'|$.

Taking moments of the above equation with respect to $\rho_m(\mathbf{c}, \varepsilon_i)$, we obtain the following system of integral equations:

$$a_m(r) = \gamma S_m(r) + \tau_{tr} J_m(r) + \tau_{int} L_m(r) + \sum_{j=1}^{5} \int_{r_0}^{\infty} K_{mj}(r, r') a_j(r')\, dr' \quad (40)$$

where the source terms S_m, J_m, and L_m are given by

$$S_m(r) = (\rho_m(\mathbf{c}, \varepsilon_i; r), 1) \quad (41)$$

$$J_m(r) = (\rho_m(\mathbf{c}, \varepsilon_i; r), (c^2 - 2)) \quad (42)$$

$$L_m(r) = (\rho_m(\mathbf{c}, \varepsilon_i; r), (\varepsilon_i - G)). \quad (43)$$

Here the scalar product is defined as

$$(\rho_m, f) = \sum_i \frac{\exp(-\varepsilon_i)}{Q_0 \pi^{3/2}} \times \int d\mathbf{c} \exp(-c^2 - |\mathbf{r} - \mathbf{r}_0|/c) \rho_m f. \quad (44)$$

Also, the kernel $K_{mj}(r, r')$ is given by

$$K_{mj}(r, r') = \frac{2r'}{\pi^{1/2} r} \int_{|r-r'|}^{(r^2-r_0^2)^{1/2} + (r'^2 - r_0^2)^{1/2}} dt/t \times \left\{ \sum_i \frac{\exp(-\varepsilon_i)}{Q_i} \int_0^\infty c\, dc \exp(-c^2 - t/c) \times \rho_m(\mathbf{c}, \varepsilon_i; r) \psi_j(\mathbf{c}, \varepsilon_i; r') \right\}. \quad (45)$$

The source terms and kernels are further simplified into forms suitable for computation (see Appendix A). The functions $a_m(r)$ can be written as

$$a_m(r) = \gamma P_m(r) + \tau_{tr} Q_m(r) + \tau_{int} R_m(r) \quad (46)$$

where the functions $P_m(r)$, $Q_m(r)$ and $R_m(r)$ are determined from the integral equations

$$P_m(r) = S_m(r) + \int_{r_0}^{R_\infty} \sum_{j=1}^{5} K_{mj}(r, r') P_j(r')\, dr' \quad (47)$$

$$Q_m(r) = J_m(r) + \int_{r_0}^{R_\infty} \sum_{j=1}^{5} K_{mj}(r, r') Q_j(r')\, dr' \quad (48)$$

$$R_m(r) = L_m(r) + \int_{r_0}^{R_\infty} \sum_{j=1}^{5} K_{mj}(r, r') R_j(r')\, dr'. \quad (49)$$

Integral equations (47)–(49) can be solved numerically. Once $P_m(x)$, $Q_m(x)$, and $R_m(x)$ are known, the complete solution of equation (40) can be constructed through determination of γ, τ_{tr}, and τ_{int} by using equations (39), (23)–(26) and (46). Using these equations one finds that

$$\begin{pmatrix} \gamma \\ \tau_{tr} \\ \tau_{int} \end{pmatrix} = \tau \mathscr{D} - 1 \begin{pmatrix} 0 \\ \alpha_{tr} \\ \alpha_{int} \end{pmatrix} \quad (50)$$

The elements of matrix $\mathscr{D}$, given in Appendix B, depend on the solutions $P_m(r)$, $Q_m(r)$ and $R_m(r)$.

NUMERICAL RESULTS AND DISCUSSION

The numerical computations associated with the problem were carried out on the UMC College of Engineering's VAX 11/780 and HARRIS-800 computers. The integral equations (45)–(47), were transformed into forms suitable for numerical calculation. These equations were solved using the 81-point Gauss–Kronrod quadrature and the numerical method given in refs. [10–13]. During the course of obtaining the numerical results of this problem several different approaches were tried. We first employed the 41-point Gauss–Kronrod quadrature for the numerical integrations involved. Most of these integrals have the lower limit of r_0 and an upper limit of infinity. A suitable truncation value, R_∞, was sought to replace the upper limit. The approach failed to yield the necessary steady-state constant heat flow through the surface of the concentric reference spheres with radius r extending from $r_0 < r < R_\infty$. Next we decided to reduce the problem to the monatomic gas case, a much simpler numerical problem. However, we obtained similar results; there was variation in the total heat transfer across the surface of the sphere of radius r, which violates the assumption of steady-state heat flow. After analyzing the intermediate results we noticed some inaccuracy in the calculation of the source term integrals, especially for small values of r_0. This error was eliminated by using the DCADRE routine (one of the IMSL library routines) for integration instead of Gauss quadrature. The other change made was the use of the 81-point quadrature instead of 41-point quadrature. This approach significantly improved the results. These results can still be improved by using higher order quadratures. Use of a 112-point quadrature on both the VAX 11/780 and HARRIS-800 computers was not possible due to lack of sufficient memory.

We have calculated the heat transfer ratio, density, and translational and internal temperature profiles using physical properties of four different gases: air (at 424 K), CO_2 (400 K), SO_2 (400 K), and N_2 (424 K) with inverse Knudsen numbers ranging from 0.001 to 10. Each of the parameters Z, f_t and G for air, CO_2, and SO_2 were obtained from ref. [14], and for N_2 were obtained from ref. [15]. Figure 1 presents the heat transfer ratio for N_2 gas with $\alpha_{tr} = \alpha_{int} = 1$, and $\alpha_{tr} = \alpha_{int} = 0.5$. Tables 1 and 2 list the moments $a_1(r)$ through $a_5(r)$ for N_2 gas with an inverse Knudsen

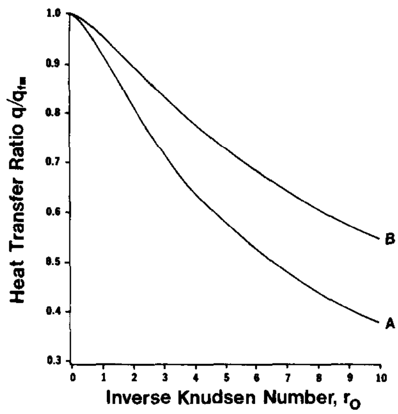

FIG. 1. Heat transfer ratio vs inverse Knudsen number, N_2 gas: (A) $f_t = 1.96$, $G = 1.0$, $Z = 5.08$, $\alpha_{tr} = 1$, $\alpha_{int} = 1$; (B) $f_t = 1.96$, $G = 1.0$, $Z = 5.08$, $\alpha_{tr} = 0.5$, $\alpha_{int} = 0.5$.

number $r_0 = 1.0$, for the case of complete accommodation (i.e. $\alpha_{tr} = \alpha_{int} = 1$ and $\alpha_{tr} = \alpha_{int} = 0.5$, respectively). Tables 3 and 4 list the translational, internal and total heat transfer, and heat transfer ratio for N_2 gas with $r_0 = 1.0$ and $\alpha_{tr} = \alpha_{int} = 1.0$, using 81-point and 41-point quadrature, respectively. Comparison of these two tables indicates a significant improvement in the results by use of the 81-point quadrature.

Several observations can be made from the results of this problem.

(1) Varying the internal accommodation coefficient α_{int} has a smaller effect on the heat transfer ratio than varying the translational accommodation coefficient α_{tr}.

(2) As the values of α_{tr} and α_{int} decrease, the heat

Table 1. $a_1(r)$ through $a_5(r)$ profiles for N_2 gas: $r_0 = 1$; $\alpha_{tr} = 1$; $\alpha_{int} = 1$

r	$a_1(r)$	$a_2(r)$	$a_3(r)$	$a_4(r)$	$a_5(r)$
1.003	−0.363E+00	0.815E+00	0.541E+00	0.513E+00	0.255E+00
1.145	−0.252E+00	0.514E+00	0.344E+00	0.393E+00	0.196E+00
1.512	−0.158E+00	0.294E+00	0.199E+00	0.225E+00	0.112E+00
2.095	−0.996E−01	0.172E+00	0.118E+00	0.117E+00	0.578E−01
2.880	−0.632E−01	0.108E+00	0.740E−01	0.622E−01	0.302E−01
3.848	−0.445E−01	0.720E−01	0.493E−01	0.350E−01	0.166E−01
4.976	−0.315E−01	0.506E−01	0.344E−01	0.210E−01	0.972E−02
6.237	−0.229E−01	0.368E−01	0.249E−01	0.134E−01	0.606E−02
7.600	−0.163E−01	0.274E−01	0.184E−01	0.905E−02	0.401E−02
9.032	−0.126E−01	0.207E−01	0.139E−01	0.641E−02	0.290E−02
10.500	−0.952E−02	0.159E−01	0.106E−01	0.475E−02	0.205E−02
11.970	−0.720E−02	0.122E−01	0.814E−02	0.365E−02	0.157E−02
13.400	−0.544E−02	0.941E−02	0.629E−02	0.291E−02	0.125E−02
14.760	−0.409E−02	0.727E−02	0.484E−02	0.240E−02	0.103E−02
16.020	−0.305E−02	0.561E−02	0.373E−02	0.203E−02	0.872E−03
17.150	−0.225E−02	0.433E−02	0.287E−02	0.177E−02	0.765E−03
18.120	−0.163E−02	0.333E−02	0.219E−02	0.158E−02	0.690E−03
18.900	−0.117E−02	0.257E−02	0.167E−02	0.145E−02	0.639E−03
19.490	−0.811E−03	0.197E−02	0.128E−02	0.136E−02	0.607E−03

Table 2. $a_1(r)$ through $a_5(r)$ profiles for N_2 gas: $r_0 = 1$; $\alpha_{tr} = 0.5$; $\alpha_{int} = 0.50$

r	$a_1(r)$	$a_2(r)$	$a_3(r)$	$a_4(r)$	$a_5(r)$
1.004	−0.189E+00	0.423E+00	0.282E+00	0.267E+00	0.133E+00
1.183	−0.124E+00	0.250E+00	0.168E+00	0.192E+00	0.957E−01
1.647	−0.734E−01	0.133E+00	0.909E−01	0.989E−01	0.492E−01
2.384	−0.443E−01	0.751E−01	0.516E−01	0.473E−01	0.232E−01
3.375	−0.282E−01	0.462E−01	0.317E−01	0.236E−01	0.113E−01
4.598	−0.189E−01	0.306E−01	0.209E−01	0.129E−01	0.596E−02
6.023	−0.132E−01	0.214E−01	0.145E−01	0.746E−01	0.339E−02
7.615	−0.944E−02	0.155E−01	0.104E−01	0.467E−02	0.207E−02
9.336	−0.690E−02	0.114E−01	0.765E−02	0.311E−02	0.135E−02
11.150	−0.511E−02	0.858E−02	0.573E−02	0.217E−02	0.335E−03
13.000	−0.382E−02	0.630E−02	0.434E−02	0.159E−02	0.680E−03
14.850	−0.286E−02	0.495E−02	0.330E−02	0.122E−02	0.518E−03
16.660	−0.214E−02	0.378E−02	0.252E−02	0.965E−03	0.410E−03
18.390	−0.159E−02	0.288E−02	0.192E−02	0.792E−03	0.337E−03
19.980	−0.117E−02	0.219E−02	0.146E−02	0.669E−03	0.286E−03
21.400	−0.850E−03	0.166E−02	0.110E−02	0.582E−03	0.250E−03
22.620	−0.606E−03	0.125E−02	0.824E−03	0.520E−03	0.226E−03
23.620	−0.422E−03	0.937E−03	0.613E−03	0.476E−03	0.209E−03
24.350	−0.285E−03	0.701E−03	0.454E−03	0.446E−03	0.199E−03
24.820	−0.185E−03	0.526E−03	0.336E−03	0.429E−03	0.193E−03
25.000	−0.124E−03	0.418E−03	0.264E−03	0.423E−03	0.191E−03

Table 3. Translational, internal and total heat transfer and heat transfer ratio profiles for N_2 gas (81-point quadrature) : $r_0 = 1$; $\alpha_{tr} = 1$; $\alpha_{int} = 1$

r	$a_4(r)$	$a_5(r)$	$r^2(a_4+a_5)$	q/q_{fm}
1.003	0.513E+00	0.255E+00	0.772E+00	0.913E+00
1.045	0.472E+00	0.235E+00	0.772E+00	0.912E+00
1.145	0.393E+00	0.196E+00	0.771E+00	0.912E+00
1.301	0.304E+00	0.151E+00	0.771E+00	0.911E+00
1.512	0.225E+00	0.112E+00	0.770E+00	0.910E+00
1.778	0.163E+00	0.806E−01	0.770E+00	0.909E+00
2.095	0.117E+00	0.578E−01	0.769E+00	0.908E+00
2.464	0.849E−01	0.415E−01	0.768E+00	0.907E+00
2.880	0.622E−01	0.302E−01	0.766E+00	0.906E+00
3.343	0.463E−01	0.222E−01	0.765E+00	0.904E+00
3.848	0.350E−01	0.166E−01	0.764E+00	0.902E+00
4.394	0.269E−01	0.126E−01	0.762E+00	0.900E+00
4.976	0.210E−01	0.972E−02	0.761E+00	0.899E+00
5.592	0.167E−01	0.762E−02	0.759E+00	0.897E+00
6.237	0.134E−01	0.606E−02	0.757E+00	0.895E+00
6.907	0.110E−01	0.489E−02	0.756E+00	0.893E+00
7.600	0.905E−02	0.401E−02	0.754E+00	0.891E+00
8.309	0.758E−02	0.333E−02	0.753E+00	0.890E+00
9.032	0.641E−02	0.280E−02	0.751E+00	0.888E+00
9.764	0.549E−02	0.238E−02	0.750E+00	0.887E+00
10.500	0.475E−02	0.205E−02	0.749E+00	0.885E+00
11.240	0.414E−02	0.178E−02	0.748E+00	0.884E+00
11.970	0.365E−02	0.157E−02	0.747E+00	0.883E+00
12.690	0.325E−02	0.139E−02	0.747E+00	0.882E+00
13.400	0.291E−02	0.125E−02	0.746E+00	0.882E+00
14.090	0.263E−02	0.112E−02	0.746E+00	0.881E+00
14.760	0.240E−02	0.103E−02	0.746E+00	0.881E+00
15.410	0.220E−02	0.942E−03	0.745E+00	0.881E+00
16.020	0.203E−02	0.872E−03	0.745E+00	0.881E+00
16.610	0.189E−02	0.814E−03	0.745E+00	0.881E+00
17.150	0.177E−02	0.765E−03	0.746E+00	0.881E+00
17.660	0.167E−02	0.724E−03	0.746E+00	0.881E+00
18.120	0.158E−02	0.690E−03	0.746E+00	0.882E+00
18.540	0.151E−02	0.662E−03	0.746E+00	0.882E+00
18.900	0.145E−02	0.639E−03	0.747E+00	0.882E+00
19.220	0.140E−02	0.621E−03	0.747E+00	0.883E+00
19.490	0.136E−02	0.607E−03	0.747E+00	0.883E+00
19.700	0.133E−02	0.595E−03	0.748E+00	0.883E+00
19.850	0.131E−02	0.588E−03	0.748E+00	0.884E+00
19.950	0.130E−02	0.583E−03	0.748E+00	0.884E+00
20.000	0.129E−02	0.581E−03	0.748E+00	0.884E+00

Table 4. Translational, internal and total heat transfer and heat transfer ratio profiles for N_2 gas (41-point quadrature) : $r_0 = 1$; $\alpha_{tr} = 1$; $\alpha_{int} = 1$

r	$a_4(r)$	$a_5(r)$	$r^2(a_4+a_5)$	q/q_{fm}
1.011	0.505E+00	0.251E+00	0.772E+00	0.912E+00
1.065	0.453E+00	0.226E+00	0.771E+00	0.910E+00
1.176	0.371E+00	0.185E+00	0.769E+00	0.908E+00
1.342	0.284E+00	0.141E+00	0.767E+00	0.906E+00
1.562	0.209E+00	0.104E+00	0.764E+00	0.903E+00
1.834	0.152E+00	0.750E−01	0.762E+00	0.900E+00
2.156	0.109E+00	0.538E−01	0.759E+00	0.896E+00
2.528	0.793E−01	0.388E−01	0.755E+00	0.892E+00
2.947	0.582E−01	0.282E−01	0.751E+00	0.887E+00
3.410	0.434E−01	0.208E−01	0.747E+00	0.882E+00
3.914	0.329E−01	0.156E−01	0.742E+00	0.877E+00
4.457	0.253E−01	0.118E−01	0.737E+00	0.871E+00
5.036	0.198E−01	0.912E−02	0.732E+00	0.863E+00
5.647	0.157E−01	0.714E−02	0.727E+00	0.859E+00
6.286	0.128E−01	0.568E−02	0.722E+00	0.854E+00
6.950	0.103E−01	0.458E−02	0.718E+00	0.848E+00
7.635	0.849E−02	0.375E−02	0.713E+00	0.843E+00
8.336	0.710E−02	0.310E−02	0.709E+00	0.838E+00
9.050	0.600E−02	0.261E−02	0.705E+00	0.833E+00
9.773	0.513E−02	0.221E−02	0.701E+00	0.829E+00
10.500	0.443E−02	0.190E−02	0.698E+00	0.825E+00
11.230	0.386E−02	0.165E−02	0.695E+00	0.821E+00
11.950	0.340E−02	0.145E−02	0.692E+00	0.818E+00
12.660	0.302E−02	0.129E−02	0.690E+00	0.816E+00
13.370	0.270E−02	0.115E−02	0.689E+00	0.814E+00
14.050	0.244E−02	0.104E−02	0.687E+00	0.812E+00
14.710	0.222E−02	0.947E−03	0.686E+00	0.811E+00
15.350	0.204E−02	0.870E−03	0.686E+00	0.810E+00
15.960	0.188E−02	0.806E−03	0.685E+00	0.810E+00
16.540	0.175E−02	0.752E−03	0.685E+00	0.810E+00
17.090	0.164E−02	0.708E−03	0.686E+00	0.810E+00
17.590	0.155E−02	0.670E−03	0.686E+00	0.811E+00
18.050	0.147E−02	0.639E−03	0.687E+00	0.812E+00
18.470	0.140E−02	0.614E−03	0.688E+00	0.813E+00
18.840	0.135E−02	0.593E−03	0.689E+00	0.814E+00
19.170	0.130E−02	0.577E−03	0.689E+00	0.815E+00
19.440	0.126E−02	0.564E−03	0.690E+00	0.816E+00
19.660	0.123E−02	0.554E−03	0.691E+00	0.817E+00
19.820	0.121E−02	0.547E−03	0.692E+00	0.818E+00
19.930	0.120E−02	0.542E−03	0.693E+00	0.819E+00
19.990	0.120E−02	0.540E−03	0.694E+00	0.820E+00

flux ratio increases, even though the total heat flow decreases.

We have solved the boundary value problem of heat transfer in a rarefied polyatomic gas from a sphere, and provided results for the quantities of interest (i.e. density translational and internal temperature perturbations, and non-dimensional translational and internal radial heat flux). These results are of fundamental interest in several aspects of aerosol mechanics, and their availability should encourage experimental measurements on single particle heat transfer in polyatomic gases under controlled surface conditions.

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APPENDIX A

The final expressions for the source terms $S_m(r)$, $J_m(r)$ and $L_m(r)$ and kernel $K_{mj}(r, r')$ in equation (40) are given by

$$S_1(r) = (1/(\pi^{1/2}r)) \int T_2((r^2-r_0^2)/t^2-1)\,\mathrm{d}t$$

$$S_2(r) = (1/(\pi^{1/2}r)) \int [T_4(t)-(3/2)T_2(t)]((r^2-r_0^2)/t^2-1)\,\mathrm{d}t$$

$$S_3(r) = 0$$

$$S_4(r) = (1/(2\pi^{1/2}r^2)) \int [T_5(t)-(5/2)T_3(t)] \times ((r^2-r_0^2)^2/t^3-t)\,\mathrm{d}t$$

$$S_5(r) = 0$$

$$J_1(r) = (1/(\pi^{1/2}r)) \int [T_4(t)-2T_2(t)]((r^2-r_0^2)/t^2-1)\,\mathrm{d}t$$

$$J_2(r) = (1/(\pi^{1/2}r)) \int [T_6(t)-(7/2)T_4(t)+3T_2(t)] \times ((r^2-r_0^2)/t^2-1)\,\mathrm{d}t$$

$$J_3(r) = 0$$

$$J_4(r) = (1/(2\pi^{1/2}r^2)) \int [T_7(t)-(9/2)T_5(t)+5T_3(t)] \times ((r^2-r_0^2)^2/t^3-t)\,\mathrm{d}t$$

$$J_5(r) = 0$$

$$L_1(r) = 0$$

$$L_2(r) = 0$$

$$L_3(r) = (G/\pi^{1/2}r) \int T_2(t)((r^2-r_0^2)/t-1)\,\mathrm{d}t$$

$$L_4(r) = 0$$

$$L_5(r) = (G/(2\pi^{1/2}r^2)) \int T_3(t)((r^2-r_0^2)^2/t^3-t)\,\mathrm{d}t$$

where the integration extends from $(r-r_0)$ to $(r^2-r_0^2)^{1/2}$

$$K_{11}(r,r') = 2(r'/\pi^{1/2}) \int (T_1/t)\,\mathrm{d}t$$

$$K_{12}(r,r') = (4/3)(r'/\pi^{1/2}r)(1-(2G/3Z)) \int (T_3-(3/2)T_1)\,\mathrm{d}t/t$$

$$K_{13}(r,r') = (4/3)(r'/\pi^{1/2}r)(1/Z) \int (T_3-(3/2)T_1)\,\mathrm{d}t/t$$

$$K_{14}(r,r') = (4/9)(1/\pi^{1/2}r)(1-G/Z) \times \int (T_4-(5/2)T_2)((r^2-r'^2)/t^2-1)\,\mathrm{d}t$$

$$K_{15}(r,r') = (4/9)(1/\pi^{1/2}r)(1/Z) \times \int (T_4-(5/2)T_2)((r^2-r'^2)/t^2-1)\,\mathrm{d}t$$

$$K_{21}(r,r') = 2(r'/\pi^{1/2}r) \int (T_3-(3/2)T_1)\,\mathrm{d}t/t$$

$$K_{22}(r,r') = (4/3)(r'/\pi^{1/2}r)(1-2G/3Z) \times \int (T_5 - 3T_3 + (9/4)T_1)\,dt/t$$

$$K_{23}(r,r') = (4/3)(r'/\pi^{1/2}r)(1/Z)\int (T_5 - (3T_3 + (9/4)T_1)\,dt/t$$

$$K_{24}(r,r') = (4/9)(1/\pi^{1/2}r)(1-G/Z) \times \int (T_6 - 4T_4 + (15/4)T_2)((r^2 - r'^2)/t^2 - 1)\,dt$$

$$K_{25}(r,r') = (4/9)(1/\pi^{1/2}r)(1/Z) \times \int (T_6 - 4T_4 + (15/4)T_2)((r^2 - r'^2)/t^2 - 1)\,dt$$

$$K_{31}(r,r') = 0$$

$$K_{32}(r,r') = 2(r'/\pi^{1/2}r)(2G/3Z)\int (T_1/t)\,dt$$

$$K_{33}(r,r') = 2(r'/\pi^{1/2}r)(1-1/Z)\int (T_1/t)\,dt$$

$$K_{34}(r,r') = (1/\pi^{1/2}r)(2G/3Z)\int T_2((r^2 - r'^2)/t^2 - 1)\,dt$$

$$K_{35}(r,r') = (1/\pi^{1/2}r)(1-F)\int T_2((r^2 - r'^2)/t^2 - 1)\,dt$$

$$K_{41}(r,r') = (r'/\pi^{1/2}r^2)\int (T_4 - (5/2)T_2)((r^2 - r'^2)/t^2 - 1)\,dt$$

$$K_{42}(r,r') = (2/3)(r'/\pi^{1/2}r^2)(1-2G/3Z) \times \int (T_6 - 4T_4 + (15/4)T_2)((r^2 - r'^2)/t^2 + 1)\,dt$$

$$K_{43}(r,r') = (2/3)(r'/\pi^{1/2}r^2)(1/Z) \times \int (T_6 - 4T_4 + (15/4)T_2)((r^2 - r'^2)/t^2 + 1)\,dt$$

$$K_{44}(r,r') = (2/9)(1/\pi^{1/2}r^2)(1-G/Z) \times \int (T_7 - 5T_5 + (25/4)T_3)((r^2 - r'^2)^2/t^3 - t)\,dt$$

$$K_{45}(r,r') = (1/3)(1/\pi^{1/2}r^2)(1/Z) \times \int (T_7 - 5T_5 + (25/4)T_3)((r^2 - r'^2)^2/t^3 - t)\,dt$$

$$K_{51}(r,r') = 0$$

$$K_{52}(r,r') = (2/3)(r'/\pi^{1/2}r^2)(G/Z)\int T_2((r^2 - r'^2)/t^2 + 1)\,dt$$

$$K_{53}(r,r') = (r'/\pi^{1/2}r^2)(1-1/Z)\int T_2((r^2 - r'^2)/t^2 + 1)\,dt$$

$$K_{54}(r,r') = (1/\pi^{1/2}r^2)(G/3Z)\int T_3((r^2 - r'^2)/t^3 - t)\,dt$$

$$K_{55}(r,r') = (1/\pi^{1/2}r^2)(1-F)\int T_3((r^2 - r'^2)/t^3 - t)\,dt.$$

The limit of integration in the above integrals are from $|\mathbf{r}-\mathbf{r}'|$ to $(r^2-r_0^2)^{1/2}+(r'^2-r_0^2)^{1/2}$ and the argument of T_n functions is $t = |\mathbf{r}-\mathbf{r}'|$. The T_n are Abramowitz functions defined by

$$T_n(x) = \int_0^\infty t^n \exp(-t^2 - x/t)\,dt.$$

APPENDIX B

The elements of matrix $\mathscr{D}$ in equation (50) are given by

$$D_{11} = 1 + \frac{2}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rP_m(r)x_{m,1}(r)\,dr$$

$$D_{12} = \frac{2}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rQ_m(r)x_{m,1}(r)\,dr$$

$$D_{13} = \frac{2}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rR_m(r)x_{m,1}(r)\,dr$$

$$D_{21} = \frac{(1-\alpha_{tr})}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rP_m(r)x_{m,2}(r)\,dr$$

$$D_{22} = 1 + \frac{(1-\alpha_{tr})}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rQ_m(r)x_{m,2}(r)\,dr$$

$$D_{23} = \frac{(1-\alpha_{tr})}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rR_m(r)x_{m,2}(r)\,dr$$

$$D_{31} = \frac{2(1-\alpha_{int})}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rP_m(r)x_{m,3}(r)\,dr$$

$$D_{32} = \frac{2(1-\alpha_{int})}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rQ_m(r)x_{m,3}(r)\,dr$$

$$D_{33} = 1 + \frac{2(1-\alpha_{int})}{r_0^2}\sum_{m=1}^{5}\int_{r_0}^{\infty} rR_m(r)x_{m,3}(r)\,dr$$

where

$$x_{m,1}(r) = ((\psi_m(\mathbf{c},\varepsilon_j;r), c^2))$$
$$x_{m,2}(r) = ((\psi_m(\mathbf{c},\varepsilon_j;r), c^2(c^2-2)))$$
$$x_{m,3}(r) = ((\psi_m(\mathbf{c},\varepsilon_j;r), c^2(\varepsilon_j - G)))$$

and the inner product is defined by

$$((\psi_m, f)) = \sum_j \frac{\exp(-\varepsilon_j)}{Q_0}\int_{r-r_0}^{(r^2-r_0^2)^{1/2}} (1-(r^2-r_0^2)/t^2)\,dt \times \int_0^\infty \psi_m(\mathbf{c},\varepsilon_j)\exp(-c^2 - t/c) f\,dc.$$

TRANSFERT THERMIQUE DANS UN GAZ RAREFIE POLYATOMIQUE—II. SPHERE

Résumé—Le modèle de Hanson–Morse de l'équation linéarisée de Wang Chang–Uhlenbeck est utilisé pour calculer le transfert de chaleur pour une particule sphérique située dans un gaz polyatomique infiniment détendu. On obtient des résultats pour le transfert de chaleur, les profils de densité et de températures interne et de translation, pour tous les degrés de raréfaction et pour des coefficients d'accommodation interne et de translation arbitraires. On étudie aussi la dépendance de ces résultats vis-à-vis du nombre de Knudsen, de l'énergie interne, du facteur d'Eucken et du nombre de relaxation de collision.

WÄRMEÜBERGANG IN EINEM VERDÜNNTEN MEHRATOMIGEN GAS AN KUGELFÖRMIGE PARTIKEL

Zusammenfassung—Es wird das Hanson–Morse-Modell der linearisierten Wang Chang–Uhlenbeck-Gleichung verwendet, um den Wärmeübergang an einem kugelförmigen Partikel zu berechnen, das sich in einem unendlich ausgedehnten mehratomigen Gas befindet. Es liegen Ergebnisse für den Wärmeübergang, die Dichte und die inneren sowie translatorischen Temperaturprofile vor für alle Verdünnungsstufen und für beliebige innere und translatorische Akkomodationskoeffizienten. Ferner wird die Abhängigkeit dieser Ergebnisse von der Knudsen-Zahl, der inneren Energie, dem Eucken-Faktor und der Kollisions–Relaxations-Zahl untersucht.

ТЕПЛООБМЕН В РАЗРЕЖЕННОМ МНОГОАТОМНОМ ГАЗЕ—II. СФЕРА

Аннотация—Модель Хансона-Морзе линеаризозанного уравнения Ванг Чанга–Уленбека применяется для расчета теплообмена от сферической частицы, находящейся в бесконечном объеме многоатомного газа. Получены результаты для теплообмена, плотности и профилей внутренней и постчпательной температуры для всех степеней разрежения и произвольных коэффициентов аккодации. Изучается зависимость получеснных разультатов от числа Кнудсена, внутренней энергии, коэффициента Эйкена и частоты столкновений.